Progress of Research on Potential Vorticity and its Inversion

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Abstract

In this paper potential vorticity (PV) and potential vorticity inversion (PVI) and their application in the cyclogenesis and mesoscale meteorological phenomena are reviewed. PV has been served as a powerful and useful dynamic tracer for the understanding of the large-scale dynamics and synoptic variations in the atmosphere and oceans. Significant progress has been made on the application of PV in recent decades there has been a substantial amount of work done on PV in a general moist atmosphere. The GMPV is defined for a real atmosphere by introducing a generalized potential temperature instead of the potential temperature or equivalent potential temperature. Such a generalization can depict the moist effect on PV anomaly in the non-uniformly saturated atmosphere.

Apart from PV, its inversion theory is also widely accepted as a useful tool in atmospheric diagnostics and numerical forecasting. If the suitable balanced models are chosen, the equations of potential vorticity inversion can be sufficient to deduce, diagnostically, all the other dynamical fields, such as winds, temperatures, geopotential heights, static stabilities, and vertical velocities, under a suitable balance condition. Several common kinds of potential vorticity inversion operators, and the associated balanced models, are introduced in this paper. It is confirmed that the potential vorticity and its inversion theory not only play an important role in studying the evolution and development of weather system, but also would give new content in the present weather forecasting system.

Key Words: potential vorticity, potential vorticity inversion, application of potential vorticity

Introduction-the brief history of potential vorticity (PV)

Large-scale fluid motions within the atmosphere and oceans are strongly influenced by both the planetary rotation and density stratification. Rotation and stratification combine to render fluid motions 'layerwise two-dimensional' with motions often substantially weaker than horizontal motions. These motions are mainly governed by a single nearly conservative scalar field, the potential vorticity(PV) because higher-frequency inertia-gravity waves and acoustic waves play only a minor role. The simplest version of PV was given by Rossby (1940) in the barotropic flow. In his paper, PV is defined as

$$PV = \zeta_a / h \tag{1}$$

where h is the depth of a material fluid column in the barotropic model, $\zeta_a = \zeta + f$ is the vertical component of the absolute vorticity. Later on, Ertel (1942) derived a general form of PV in the baroclinic flow

$$PV = \rho^{-1}\zeta_a \cdot \nabla\theta \tag{2}$$

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where ρ and θ are respectively the density and potential temperature of dry air, and ζ_a is a three-dimensional absolute vorticity vector. Another historical landmark was the introduction of isentropic potential vorticity (IPV) concept (Reed and Sanders, 1953；Reed, 1955). IPV is defined as

$$IPV = -g(\varsigma_\theta + f)(\frac{\partial\theta}{\partial p}) \tag{3}$$

The IPV is a product of the absolute vorticity and the static stability. PV derives its name from the fact that there is a potential for creating relative vorticity by changing latitude and by adiabatically changing the separation between isentropic layers. The computer-generated IPV maps were first given by Obukhov (1964) and Danielsen (1967, 1968) for the 300K, 305K and 310K isentropic surfaces. The usefulness of "IPV thinking" was discussed in Hoskins et al. (1985), in which they showed that this concept stemmed first from the invertibility of the PV field. PV describes a mass-weighted circulation, and conservation of PV suggests that a parcel may exchange stratification for circulation but is not permited across isentropic surfaces.

In recent years, PV has been increasingly used in the diagnosis of observed atmospheric behaviour, and understanding of synoptic and large-scale dynamics, atmospheric numerical simulation results, and in studies of oceanic circulations. Nevertheless, PV is not conserved when latent heat release is taken into account in the saturated moist atmosphere. Bennetts and Hoskins (1979) first generalized PV into moist potential vorticity (MPV) defined as

$$PV = \rho^{-1}\zeta_a \cdot \nabla\theta_e \tag{4}$$

by replacing θ with the equivalent potential temperature θ_e. Schubert et al. (2001) proved an annihilation of the solenoidal term in the MPV equation, thus leading to a conservation of MPV in moist adiabatic and frictionless processes. The MPV concept has extensively been used in studies of conditional symmetric instability (Emanuel 1983, 1988; Bennetts and Sharp 1982; Shutts 1990), and the generation of MPV in extratropical cyclones (Cao and Cho 1995).

In fact, the moist dynamic problems are often complex and hard to solve in the mesoscale convective system, air is neither dry nor saturated. The real case is non-uniformly saturated. For this situation, Gao et al. (2004) further the MPV concept into a generalized PV (GMPV) by replacing the potential temperature θ with the generalized potential temperature θ^*. GMPV is defined as

$$GMPV = \rho^{-1}\zeta_a \cdot \nabla\theta^* \tag{5}$$

GMPV is important advance in the study of PV dynamics in the mesoscale field, and θ^* is very useful thermodynamic variable which has been applied in Q vector and in the Richardson formula to study strong convection and instability in non-uniformly saturated atmosphere (Yang et al. 2007; Yang and Gao 2006). With an introduction of generalized potential temperature into the thermodynamic framework, we saw another approach. The tendency equation of the generalized potential vorticity can be used not only in theoretical analysis but also in practical prediction of weather systems.

Some properties of PV

Since theta is a material surface, the gradient of theta represents a layer of mass trapped in the two material surfaces. It can be seen from its various formulations that PV captures rotation nature of a layered air, whether dry or moist. Because of the atmosphere's stable stratification, $\nabla\theta$ is usually directed nearly vertically. Therefore, PV is a measure of the component of absolute spin about the vertical, including the vertical component of the Earth's rotation. More precisely, PV can be regarded as measuring the intrinsic "cyclonicity" of an air parcel, in a sense that is highly re1evant to the stratification-constrained, layered-two-dimensional motion. This is re1ated to the fact that, on each isentropic surface of the stable stratification, PV is proportional to the component of absolute vorticity precisely normal to that surface, together with the fact this component tends to increase or "spin up" when $|\nabla\theta|$ is decreased by adiabatic vertical motion, and vice versa, tending to keep the PV value of an air parcel constant. Extratropical stratospheric air has a very high intrinsic cyclonicity, in this sense, in comparison with tropospheric air. This is a key factor: for instance, in extratropical explosive cyclogenesis, one of the commonest causes is extratropical 1ower-stratospheric air descending along a sloping isentropic surface and interacting dynamically with warm, moist lower-tropospheric air (e.g. Hoskins et al. 1985; Uccellini et al. 1979; Hoskins and Berrisford 1988). This phenomenon is known as high-PV air intrusion.

The second fundamental point about PV is the idea of “invertibility”. More precisely, there is an “invertibility principle” to the effect that if (a) a suitable balance condition is imposed to eliminate gravity and inertio-gravity waves from consideration, and if (b) a suitable reference state is specified, then a knowledge of the distribution of PV on each isentropic surface, and of θ at the lower boundary, is sufficient to deduce, diagnostically, all the other dynamical fields such as winds, temperatures, pressures, and the altitudes of the isentropic surfaces (Hoskins et al. 1985).

It should be emphasized here that a local knowledge of PV does not imply a local knowledge of ψ or u, because the inversion is a global process. In particular, it depends on specifying suitable boundary conditions to make the inverse Laplacian ∇^{-2} unambiguous. Also, in this system the balance condition, on which invertibility depends, corresponds simply to the absence of sound or external gravity waves. They have been filtered out by the assumption of incompressible, nondivergent motion. Furthermore, there is a scale effect, whereby small-scale features in the PV field have a relatively weak effect on the ψ and u fields, while large-scale features have a relatively strong effect. In particular, ψ and u are to varying degrees insensitive to fine-grain structure in the PV field.

Substantially, Ertel’s theorem is a particular case of the general result expressed as follows (Ertel 1942; Obukhov 1962)

$$D(PV)/Dt = -\rho^{-1}\nabla \cdot N_Q \tag{6}$$

where the three-dimensional material derivative and the nonadvective flux or transport are defined respectively by $D/Dt = \partial/\partial t + \mathrm{u}\cdot\nabla$ and $N_Q = -\mathrm{H}\zeta_a - \mathrm{F}\times\nabla\theta$.

This is the three-dimensional velocity field in which F is the viscous or other nonconservative body force per unit mass, and H the diabatic heating rate expressed as the material rate of change of θ, i.e., $\mathrm{H} = D\theta/Dt$.

The flux form of (1), from which (1) itself can be recovered using the mass-conservation equation $\partial\rho/\partial t + \nabla\cdot\{\rho \mathrm{u}\} = 0$, expresses exact conservation and is given by:

$$\partial(\rho PV)/\partial t + \nabla\cdot \mathrm{J} = 0, \tag{7}$$

where $\mathrm{J} = \mathrm{u}\rho PV + N_P$ denotes a total PV flux. The flux form (7) expresses conservation in the most general possible sense. For instance, it is different according to whether or not the system is also mass-conserving (Haynes and McIntyre 1990).

A pursuit of the PV-chemical transport analogy to its logical conclusion leads to an expression for the flux J, showing that PV behaves as if the quasi-molecules particles of signed "PV-substance" (PVS) can be transported along isentropic surfaces, but not across them, and created or destroyed (apart from "pair production" and "mutual annihilation") only where isentropic surfaces meet boundaries (McIntyre and Norton 1990; Haynes and McIntyre 1990; Haynes and McIntyre 1987).

PV is conceptually very succinct and involves replacing the concepts of "force" and "torque" by the concept of "PV flux" or "PV transport"—or "generalized rearrangement" of PVS. The phrase "generalized rearrangement" is meant to suggest the horizontal migration of PVS particles confined to each θ-layer, allowing for dilution and concentration effects as mass enters and leaves the layer, and pair production and mutual annihilation in the tropics (Schubert et al. 1995).

Potential Vorticity Inversion

For a long time, people mainly paid attention to use potential vorticity as Lagrange tracer to study its distribution in atmosphere, they did not realize that the concept of potential vorticity could contain a lot of dynamic meanings. It is until 50s when Kleinschmidt (1957) first used the potential vorticity anomaly observed on the top of troposphere to explain the generation of cyclone, but his theory is ahead of time and people at that time focused on the baroclinic instability theory, so his research did not draw much attention and the potential vorticity inversion theory came late for about 20 years (Hoskins et al. 1985).

In 1985, Hoskins et al (1985) first described the potential vorticity inversion theory completely and clearly, and brought out three basic conditions under which the potential vorticity inversion theory could be carried out:

(1) fixed some balance conditions;

(2) adopted some reference state to describe the mass distribution of potential vorticity;

(3) to solve inversion problem in the whole research area under some appropriate boundary condition.

When the three conditions are satisfied, and if the mass distribution of potential vorticity is given, then the corresponding wind, pressure and temperature fields could be deduced. After Hoskins et al (1985) proposed the potential vorticity inversion theory, a climax of using potential vorticity and its inversion theory to study the possible dynamic mechanism of some synoptic phenomena was pushed out.

If the suitable balanced models are chosen, the equations of potential vorticity inversion can be sufficient to deduce, diagnostically, all the other dynamical fields, such as winds, temperatures, geopotential heights, static stabilities, and vertical velocities, under a suitable balance condition. Several common kinds of potential vorticity inversion operators, and the associated balanced models, are introduced as follows:

Inversion of isolated potential vorticity with a circular axisymmetric structure

In theory researches, an important problem is to study the dynamic characteristic of isolated potential vorticity abnormal area (could be positive or negative) which located at stratified stable reference atmosphere. Assume that potential vorticity abnormal area has a circular axisymmetric structure, thus we can also assume that the flow field has a circular axisymmetric structure because the inversion operator is a smooth Laplacian operator. If we choose cylindrical coordinate, the direction of velocity could only be along tangent line, this problem satisfied the balance of gradient wind exactly. For choosing potential vorticity, we can choose different ones under different approximate conditions, such as the shallow water model, original equation model and so on. In the following we will introduce the conclusion which made by Kleischmidt et al (1957):

(1) the area of high potential vorticity brings out positive circulation, while negative circulation was brought out by the low one;

(2) the excited flow field extends towards the abnormal area up and down, the extended magnitude is decided by the Rossby height;

(3) the hydrostatic stability and absolute vorticity are large in the area of high potential vorticity, but in the opposite way in the area of low potential vorticity;

(4) the up and down hydrostatic stability decrease in the area of high potential vorticity, but in the opposite way in the area of low potential vorticity.

Here another conclusion which is confirmed by many observations is: the high potential vorticity which moves forward is like a “vacuum cleaner” for the atmosphere below it, the absorbed front air moves upward, and compresses rear air to make downward movement.

Quasi-geostrophic Potential Vorticity (qg) Inversion

The quasi-geostrophic model plays an important roles in atmosphere science and numerical forecasting, thus obviously it is necessary to study quasi-geostrophic potential vorticity inversion. Trough introducing geostrophic streamfunction ψ, the relationship between ψ and qg can be deduced as:

$$£g\psi = qgf \qquad (8)$$

Here, £g is linear and elliptical operator. The inversion equation is linear, so the inversion of quasi-geostrophic potential vorticity is mathematically simple (linear superposition principle) and it is convenient for theoretical investigation. But big error will appear when Rossby number becomes large, even at this time it can provide useful qualitative information (Kuo et al. 1991).

Connections with principles derived from synoptic experience are indicated, such as the 'PVA rule' concerning positive vorticity advection on upper air charts, and the role of disturbances of upper air origin, in combination with low-level warm advection, in triggering latent heat release to produce explosive cyclonic development. In all cases it is found that time sequences of isentropic potential vorticity and surface potential temperature charts - which succinctly summarize the combined effects of vorticity advection, thermal advection, and vertical motion without requiring explicit knowledge of the vertical motion field- lead to a very clear and complete picture of the dynamics. This picture is remarkably simple in many cases of real meteorological interest. It involves, in principle, no sacrifices in quantatative accuracy beyond what is inherent in the concept of balance, as used for instance in the initialization of numerical weather forecasts.

Piecewise Potential Vorticity Inversion

The method of piecewise potential vorticity inversion is a diagnosis method with rich dynamic meanings. Here the field of potential vorticity was divided into the isolated potential vorticity abnormal area and ambient environmental field, then we can inverse the abnormal area of potential vorticity, analyze its contribution to the whole flow field and the interaction between them. In another word, if there were several potential vorticity anomalies when some weather phenomenon occurred, we could use the method of piecewise potential vorticity inversion to diagnose which area of potential vorticity anomaly is the main reason which caused this phenomenon, thus we can deduce the dynamic mechanism in this phenomenon. But there is a problem that the sum of flow fields of piecewise potential vorticity inversion is not all equal to the flow field of whole inversion. In the language of mathematics: if the inversion equation is linear differential equation (such as the quasi-geostrophic approximation), then the superposition principle satisfied, the sum of flow fields of piecewise potential vorticity inversion is equal to the flow field of whole inversion; if it is nonlinear equation, then the sum of flow fields of piecewise potential vorticity inversion could not be equal to the flow field of whole inversion. Sometimes in order to assure the equality, the situation that piecewise flow field could not possibly be inversed by the corresponding potential vorticity will occur. So except for the linear inversion problem, piecewise potential vorticity inversion could not be precise in common situations.

For the nonlinear inversion equation of Ertel potential vorticity under the condition of Charney balance and hydrostatic equilibrium, Davis (1992) put forward four kinds of inversion methods: Full Linear (FL), Truncated Linear (TL), Subtraction Technique (ST) and Addition Method (AM). Among them, the first two methods are linear, and the others are nonlinear. The shortcoming of nonlinear piecewise inversion is that the sum of flow fields of piecewise potential vorticity inversion could not be equal to the flow field of whole inversion, but it retains the nonlinear terms with important meanings in the inversion equation.

Piecewise potential vorticity inversion is widely accepted as a useful tool in atmospheric diagnostics. This method is thought to quantify the instantaneous interaction at a distance of anomalies of potential vorticity separated horizontally and/or vertically. Doubts with respect to the dynamical justification of PPVI are formulated. In particular, it is argued that the tendency of the inverted streamfunction must be determined in order to quantify far-field effects. Elementary tests of PPVI are conducted to clarify these points (Egger, J., 2008).

It is concluded that PPVI with additional tendency calculations poses and solves a specific problem by retaining observed PV anomalies in one subdomain and removing them in others. The usefulness of the results with regard to the diagnosis of the observed state depends strongly on the flows considered and on the partitions chosen, which must comply with a simple rule (Egger, J., 2008).

The application of Potential Vorticity Inversion

The application of potential vorticity inversion theory can be divide into two stages: The first stage began from the early 80s, Hoskins et al. (1985) proposed that the adiabatic and frictionless atmosphere has tendency of doing two-dimensional motion along the isentropic surface as well as the rich dynamics connotation of potential vorticity (potential vorticity inversion theory), thus the isentropic maps of potential vorticity is a very useful tool to represent dynamical processes in the atmosphere. By use of the above theories, clear physical images and interpretations of these issues can be given: (a) the structure, origin and persistence of cutoff cyclones and blocking anticyclones, (b) the physical mechanisms of Rossby wave propagation, baroclinic instability, and barotropic instability, and (c) the spatially and temporally nonuniform way in which such waves and instabilities may become strongly nonlinear, as in an occluding cyclone or in the formation of an upper air shear line. The second stage began from the early 90s, Davis (1992) proposed the method of piecewise potential vorticity inversion. Making use of the conservation of potential vorticity, potential vorticity anomalies which are caused by non-conservative processes can be separated. By use of this method, the contribution to wind and pressure fields can be diagnosed, thus the reasons and essential characteristics of some phenomena can be inferred. The researches of this aspect mainly use the quasi-geostrophic potential vorticity and Ertel potential vorticity and it mainly focus on the following two fields: Firstly, using the observation data and theoretical model, the dynamic mechanism of cyclogenesis and secondary circulation in front can be quantitatively diagnosed; secondly, using the potential vorticity error from observation data and model forecasting, dynamically consistent wind, pressure and temperature fields can be inversed, and the initial field can be modified by adding these fields into the original one, in order to improve the accuracy of short-term forecasting (Huo Z , D Zhag , J Gyakum, 1993; Fehlmann R , H C Davies, 1997). Conclusively, applications of both these aspects are very successful, especially for the diagnostic research on dynamic mechanism of cyclone.

Discussion and Conclusion

PV is at the core of balanced atmospheric flow dynamics. It is conservative in the adiabatic and frictionless conditions; it is easy to express its behaviors by analysis of its advection behaviors. More importantly, in a balanced system, PV has invertibility, and so it can be used to derive the wind field. All these have brought great convenience to the study of large-scale dynamics problems. The GMPV is derived for a real atmosphere (neither totally dry nor saturated) by introducing a generalized potential temperature instead of the potential temperature or equivalent potential temperature. Such a generalization can depict the moist effect on PV anomaly in the non-uniformly saturated atmosphere.

Apart from PV, its inversion theory also plays an important role in studying the evolution and development of weather system, but doubts with respect to the inversion of potential vorticity are formulated: How to define the average state of potential vorticity? Different anomaly areas will be separated with different average states, thus the reliability of dynamical diagnosis will be reduced because of the non-uniqueness; all the inversions of Ertel potential vorticity are carried out by the Charney balance, but it is not suitable for the mesoscale convective systems, such as front, squall line and so on; the role of friction cannot be neglected as to the anomaly of potential vorticity nearby the ground, how to think about this influence on the inversion is another problem.

It is confirmed that the potential vorticity and its inversion theory not only play an important role in studying the evolution and development of weather system, but also would give new content in the present weather forecasting system (McIntyre M E., 1994), which shown as two points below:

(1) It would extend the diagnosis range for the cyclone, study the generation and development mechanism of various cyclones and secondary circulation in front, reveal the interaction with potential vorticity anomaly which caused by different reasons (such as the decrease of top troposphere, diabatic heating and cooling, the change of potential temperature near ground and so on). For the study of tropical cyclone, some people have already begun to bring new balance conditions (M¨oller J D, Smith R K.,1994; M¨oller J D , S C Jones., 1998).

(2) With the improvement of data assimilation and observations, it will be possible to make the high resolution and dynamic consistent IPV image. With the help of dynamic IPV image and people’s highly developed vision system and the artificial intelligent technology, weather changes could be forecasted more precisely in the future.

There are many domestic papers using potential vorticity to diagnose cyclone and storm, but the research about potential vorticity inversion is not carried out yet. China is a country with all kinds of cyclones occurring frequently, so it is essential to carry out the dynamic diagnosis of piecewise potential vorticity inversion.

References

Bennetts, D.A., and B.J. Hoskins, 1979: Conditional symmetric instability –A possible explanation for frontal rainbands. Quart. J. Roy. Meteor. Soc., 105, 945-962.

Bennetts, D.A., and J.C. Sharp, 1982: The relevance of conditional symmetric instability to the prediction of mesoscale frontal rainbands. Quart. J. Roy. Meteor. Soc., 108, 595-602.

Cao, Z., and H.-R. Cho, 1995: Generation of moist potential vorticity in extratropical cyclones. J. Atmos. Sci., 52, 3263-3281.

Danielsen, E. F., 1967: Transport and diffusion of stratospheric radioactivity based on synoptic hemispheric analyses of potential vorticity. Dept. of Met. Penn. State Univ., Report NYO 3317-3

Danielsen, E. F., 1968: Stratospheric-tropospheric exchange based on radioactivity, ozeone and potential vorticity. J. Atmos. Sci., 25, 502-518.

Davis Ch A. Piecewise potential vorticity inversion[J] . **J Atmos Sci , 1992** , 49 : 1397 – 1411.

Egger, J., 2008: Piecewise Potential Vorticity Inversion: Elementary Tests. J. Atmos. Sci., 65, 2015–2024.

Emanuel, K.A., 1983: The Lagrangian parcel dynamics of moist symmetric instability. J. Atmos. Sci., 40, 2368-2376.

Emanuel, K.A., 1988: Observational evidence of slantwise convective adjustment. Mon. Wea. Rev., 116, 1805-1816.

Ertel, H., 1942: Ein Neuer hydrodynamischer Wirbelsatz. Met. Z., 59, 272-281.

(English translation in W.Schroder: Geophysical Hydrodynamics and Ertel's potential vorticity (Selected Papers of Hans Ertel), pp. 33-40. Interdivisional Commission of History, No.12, International Association of Geomagnetism and Aeronomy, Hechelstrasse 8, D-2820 Bremen-Ronnebeck, Germany, 218 pp (1991).

Fehlmann R , H C Davies. Misforecasts of synoptic systems : Diagnosis via PV-retrodiction [J] . **Mon Wea Rev , 1997** , 125: 2247 – 2264.

Gao, S.T., X. R. Wang, and Y.S. Zhou, 2004: Generation of generalised moist potential vorticity in a frictionless and moist adiabatic flow, Geophys. Res. Lett., 31, L12113, doi:10.1029/2003GL019152.

Haynes P.H., and M.E. McIntyre, 1987: On the evolution of vorticity and potential vorticity in the presence of diabatic heating and frictional or other forces. J. Atmos. Sci., 44, 828-841.

Haynes P.H., and M.E. McIntyre, 1990: On the conservation and impermeability theorems for potential vorticity. J. Atmos. Sci., 47, 2021-2031.

Hoskines, B.J., M.E. McIntyre and A.W. Robertson, 1985: On the use and significance of isentropic potential-vorticity maps. Quart. J. Roy. Meteorol. Soc., 111, 877-946. Also 113, 402-404.

Hoskins B.J. and P. Berrisford, 1988: A potential-vorticity perspective of the storm of 15-16 October 1987, Weather, 43, 122-129.

Huo Z , D Zhag , J Gyakum. Application of potential vorticity inversion to improving the numerical prediction of the March 1993 Superstorm[J] . Mon Wea Rev , 1998 , 126 : 424 – 436.

Johnny C. L. Chan, Francis M. F. Ko, and Ying Man Lei, 2001: Relationship between potential vorticity tendency and tropical cyclone motion. J. Atmos. Sci., 59, 1337-1336.

Kleinschmidt E. Handbuch der Physik [M] . In : Eliassen A ,Kleinschmidt E eds. Dynamic Meteorology. Germany :Speringer , 1957. 48 : 112 – 129

Kuo Y H , M A Shapiro , E G Donall E G. The interaction between baroclinic and diabatic processes in a numerical simulation of a rapidly intensifying extratropical marine cyclone [J] . Mon Wea Rev , 1991 , 119 : 368 – 384

McIntyre M E. Numerical weather prediction: An updated vision of the future[C] . Vol. I. Proceedings of an international symposium. S. Gr<nÜas , M. A. Shapiro (Hrsg.) , Bergen , Norwegen , 1994 , 275 – 286

McIntyre, M.E. and W.A. Norton: Dissipative wave-mean interactions and the transport of vorticity or potential vorticity. J.Fluid Mech. 212 (G.K. Batchelor Festschrift Issue), 403-435 (1990); see also Corrigendum, 220, 693 (1990).

McIntyre, M.E., and W.A. Norton, 2000: Potential Vorticity Inversion on a Hemisphere. J. Atmos. Sci., 57, 1214–1235.

M¨oller J D , S C Jones. Potential vorticity inversion for tropical cyclones using the asymmetric balance theory[J] . J Atmos Sci , 1998 , 55 : 259 – 282.

M¨oller J D , Smith R K. The development of potential vorticity in a hurricane like vortex [J] . Quart J R Meteor Soc , 1994 , 120 : 1255 – 1265.

Obukhov, A.M.: On the dynamics of a stratified liquid. Dokl.Akad.Nauk SSSR, 145 (6), 1239-1242. English transl. In Soviet Physics-Doklady 7, 682-684 (1962).

Obukhov, A. M., 1964: Adiabatic invariants of atmospheric processes. Meteorogiyi gidrologiya, 2: 3-9.

Reed, R. J. and Sanders, F., 1953: An investigation of the development of a mid-tropospheric frontal zone and its associated vorticity field. Ibid., 10:338-349.

Reed, R. J., 1955: A study of characteristic type of upper-level frontogenesis. J. Met., 12:226-237.

Rossby, C. G. 1940: Planetary flow patterns in the atmosphere. Quart. J. Roy. Meteor. Soc., 66(Suppl), 68-87.

Schubert, W. H., P. E. Ciesielski, C. Lu, and R. H. Johnson, 1995: Dynamical adjustment of trade wind inversion layer. J. Atmos. Sci., 52, 2941-2952.

Schubert H. W., S. A. Hausman, M Garcia, K. V. Ooyama, H Kuo, 2001: Potential vorticity in a moist atmosphere. J. Atmos. Sci., 58, 3148-3157.

Shutts, G.J., 1990: Dynamical aspects of the October storm, 1987: A study of a successful fine-mesh simulation. Quart.J.Roy.Meteor.Soc., 116, 1315-1347.

Thorpe A J . Diagnosis of balanced vortex structure using potential vorticity[J] . J Atmos Sci , 1985 , 42 : 397 – 406

Uccellini, L.W., D.Keyser, K.F.Brill and C.H.Wash: The Presidents' Day cyclone of 1819 February 1979: Influence of upstream trough amplification and associated tropopause folding on rapid cyclogenesis. Mon.Wea.Rev., 113, 962-988 (1985).

Yang, S., and S.T. Gao, 2006: Modified Richardson number in non-uniform saturated moist flow, Chin. Phys. Lett , 23, 3003-3006.

Yang, S., S.T. Gao, and D.H. Wang, 2007: Diagnostic analyses of the ageostrophic Q vector in the non-uniformly saturated, frictionless, and moist adiabatic flow, J. Geophys. Res., 112, D09114, doi:10.1029/2006JD008142.